MATH 103B – Discussion Worksheet 5
May 11, 2023

Announcements:

1. Midterm 2 is on Wednesday May 17 during class time.
2. There will be NO discussion session next Thursday May 18.
3. The TAs will hold additional office hours to be announced via Canvas.

Topics: UFDs and PIDs (Judson 18.2); Field extensions (Judson 21.1)
Let R be an integral domain.

Problem 1.

1. State the definition of an *irreducible* element in R, and give two concrete examples.
2. State the definition of a *prime* element in R, and give two concrete examples.
3. State the definition of a *principal ideal domain* (PID), and give two examples. Is a PID always a UFD?
4. State the definition of a *unique factorization domain* (UFD), and give two examples of UFDs that are *not* PIDs.
5. Unique factorization holds in a UFD up to which two conditions? Give one example for each.

Problem 2.

1. Prove or disprove: If $x \in R$ is irreducible, then x is prime.
2. Prove or disprove: If $x \in R$ is prime, then x is irreducible.
3. Does the above statements hold for a PID?
4. Does the above statements hold for a UFD?

Problem 3. Compute the degree of the following field extensions (i.e. the degree of the top field as a vector space over the base field).

1. $\mathbb{Q}(\sqrt{5})/\mathbb{Q}$.
2. $\mathbb{Q}(\sqrt[3]{5})/\mathbb{Q}$.
3. $\mathbb{Q}(\sqrt{2}+\sqrt{3})\mathbb{Q}$.